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Magnetized Bianchi Type III Massive String Cosmological Models in General Relativity

Hassan Amirhashchi · Hishamuddin Zainuddin

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Abstract The present study deals with Bianchi type III string cosmological models with magnetic field. The magnetic field is assumed to be along z direction. Therefore F_{12} is only the non-vanishing component of electromagnetic field tensor F_{ij}. The expansion (θ) in the model is assumed to be proportional to the shear (σ). To get determinate solution in term of cosmic time, we have solved the fields equations in two cases (i) Reddy and (ii) Nambu string. The physical and geometrical behaviour of these models is discussed.

Keywords LRS Bianchi type III models · Magnetic field · Massive string

1 Introduction

One of the outstanding problems in cosmology today is developing a more precise understanding of structure formation in the universe, that is, the origin of galaxies and other large-scale structures. Existing theories for the structure formation of the Universe fall into two categories, based either upon the amplification of quantum fluctuations in a scalar field during inflation, or upon symmetry breaking phase transition in the early Universe which leads to the formation of *topological defects* such as domain walls, cosmic strings, monopoles, textures and other “hybrid” creatures. Cosmic strings play an important role in the study

H. Amirhashchi (✉)
Physics Department, Mahshahr Islamic Azad University, Mahshahr, Iran
e-mail: h.amirhashchi@mahshahriau.ac.ir

H. Amirhashchi
Physics Department, Universiti Putra Malaysia, 43400 UPM, Serdang, Selangor D.E, Malaysia

H. Zainuddin
Laboratory of Computational Sciences and Informatics, Institute for Mathematical Research, Universiti Putra Malaysia, 43400 UPM, Serdang, Selangor D.E, Malaysia
e-mail: hisham@putra.upm.edu.my

H. Zainuddin
Physics Department, Universiti Putra Malaysia, 43400 UPM, Serdang, Selangor D.E, Malaysia

of the early universe. Cosmic strings are topologically stable objects which might be found during a phase transition in the early universe [1]. These arise during the phase transition after the big bang explosion as the temperature goes down below some critical temperature as predicted by grand unified theories [1–6]. It is believed that cosmic strings give rise to density perturbations which lead to the formation of galaxies [7]. These cosmic strings have stress-energy and couple to the gravitational field, it may be interesting to study the gravitational effects that arise from strings.

The general relativistic treatment of strings was initiated by Letelier [8, 9], and Stachel [10]. Letelier [8] has obtained the solution to Einstein's field equations for a cloud of strings with spherical, plane and cylindrical symmetry. Then, in 1983, he solved Einstein's field equations for a cloud of massive strings and obtained cosmological models in Bianchi I and Kantowski-Sachs space-times. Benerjee et al. [11] have investigated an axially symmetric Bianchi type I string dust cosmological model in presence and absence of magnetic field. The string cosmological models with a magnetic field are also discussed by Chakraborty [12]. Tikekar and Patel [13] have discussed some Bianchi type VI_0 string models with and without magnetic fields. Tikekar and Patel [14], following the techniques used by Letelier and Stachel, obtained some exact Bianchi III cosmological solutions of massive strings in presence of magnetic field and also with vanishing magnetic field. Maharaj et al. [15] investigated the integrability of cosmic string in Bianchi type III space-time using a symmetry analysis and generalized the previous solutions obtained by Tikekar and Patel [14]. Patel and Maharaj [16] investigated stationary rotating world model with magnetic field. Ram and Singh [17] obtained some new exact solutions of string cosmology with and without a source free magnetic field for Bianchi type I space-time in the different basic form considered by Carminati and McIntosh [18]. Exact solutions of string cosmology for Bianchi type II, VI_0, VIII and IX space-times have been studied by Krori et al. [19] and Wang [20]. Singh and Singh [21] investigated string cosmological models with magnetic field in the context of space-time with G_3 symmetry. Singh [22] has also studied string cosmological models with electromagnetic field in Bianchi type II, VIII and IX space-times. Singh [23] also obtained static models of cosmic strings in general relativity. Lidsey, Wands and Copeland [24] have reviewed aspects of super string cosmology with the emphasis on the cosmological implications of duality symmetries in the theory. Recently Yadav et al. [25] have studied some Bianchi type I viscous fluid string cosmological models with magnetic field. Wang [26–29] has discussed LRS Bianchi type I and Bianchi type III cosmological models for a cloud string with bulk viscous fluid. Baysal et al. [30] have investigated the behavior of a string in the cylindrically symmetric inhomogeneous universe. Bali et al. [31–34] have obtained Bianchi types I and IX string cosmological models in general relativity. Yavuz et al. [35] have examined charged strange quark matter attached to the string cloud in the spherical symmetric space-time admitting one-parameter group of conformal motion. Recently Kaluza-Klein cosmological solutions are obtained by Yilmaz [36] for quark matter coupled to the string cloud in the context of general relativity. Reddy [37, 38], Reddy and Naidu [39], Reddy et al. [40, 41], Rao et al. [42–45], Pradhan [46, 47], Pradhan and Mathur [48], Pradhan et al. [49–51], Amirhashchi and Zainuddin [52], Tripathi et al. [53, 54] have studied string cosmological models in different contexts. Recently, Amirhashchi and Zainuddin [55] have obtained Bianchi type III strings cosmological models for stiff and anti stiff fluids.

The occurrence of magnetic fields on galactic scale is well-established fact today, and their importance for a variety of astrophysical phenomena is generally acknowledged. Several authors (Zeldovich [56], Harrison [57], Misner et al. [58], Asseo and Sol [59], Puditzand Silk [60], Kim et al. [61], Perley and Taylor [62] Kronberg et al. [63], Wolfe et al. [64], Kulsrud et al. [65] and Barrow [66] have pointed out the importance of magnetic field in different context. As a natural consequences, we should include magnetic fields in the energy-momentum tensor of the early universe. The string cosmological models with a magnetic field are also discussed by Banerjee et al. [11], Chakraborty [12], Tikekar and Patel [13, 14], Patel and Maharaj [16], Singh and Singh [21], Bail et al. [67, 68], Pradhan et al. [69–73]. Motivated the situations discussed above, in this paper, we shall focus upon the problem of establishing a formalism for studying the massive string in Bianchi III space-time. In this paper, we have investigated a new and general solution for Bianchi type-III cosmological model for a cloud of strings and electromagnetic field which is different from the other solutions. The paper is organized as follows. The metric and the field equations are presented in Sect. 2. In Sect. 3, we deal with solution of the field equations with cloud of strings and electromagnetic field. In Sect. 3.1 the field equations have been solved for case of Reddy string. We describe some physical and geometric properties of the model in Sect. 3.1.1. In Sect.3.1.2 we have given solution in absence of electromagnetic field which has been followed by some physical and geometric properties of the model. In Sect. 3.2 the field equations have been solved for case of Nambu or geometric string. We describe some physical and geometric properties of the model in Sect. 3.2.1. In Sect. 3.2.2 we have given solution in absence of electromagnetic field which has been followed by some physical and geometric properties of the model. Finally, in Sect. 4, concluding remarks are given.

2 The Metric and Field Equations

We consider the Bianchi type III metric in the form

$$ds^2 = -dt^2 + A^2 dx^2 + B^2 e^{-2\alpha x} dy^2 + C^2 dz^2, \tag{1}$$

where A, B and C are functions of t only. The energy momentum tensor for a cloud of strings with perfect fluid distribution and electromagnetic field is taken as

$$T_i^j = (p+\rho)u_i u^j + p g_i^j - \lambda x_i x^j + E_i^j, \tag{2}$$

where u_i and x_i satisfy conditions

$$u^i u_i = -x^i x_i = -1, \quad u^i x_i = 0, \tag{3}$$

p is the isotropic pressure, ρ is the proper energy density for a cloud string with particles attached to them, λ is the string tension density, u^i the four-velocity of the particles, and x^i is a unit space-like vector representing the direction of string.

In a co-moving coordinate system, we have

$$u^i = (0, 0, 0, 1), \quad x^i = \left(0, 0, \frac{1}{C}, 0\right). \tag{4}$$

Here E_i^j is the electromagnetic field given by

$$E_i^j = \frac{1}{4\pi}\left[g^{lm} F_{il} F_{jm} - \frac{1}{4} F_{lm} F^{lm} g_{ij}\right]. \tag{5}$$

We assume that magnetic field is in the x-y plane. Thus, the current is flowing along z-axis. Therefor, F_{12} is the only non-vanishing component of electromagnetic field tensor (F_{ij}). Subsequently Maxwell equation

$$F_{ik,l}+F_{kl,i}+F_{li,k}=0 \quad \text{and} \quad \left[F^{ik}(-g)^{\frac{1}{2}}\right]_{,k}=0, \tag{6}$$

lead to

$$F_{12}=ke^{-\alpha x}, \tag{7}$$

where k is a constant so the magnetic field depends upon the space coordinate x only. From (4), (5) and (6), it follows that $F_{14}=0$. Now the non-vanishing components of E_{ij} corresponding to the line-element (1) are given by

$$E_1^1=E_2^2=\frac{1}{8\pi}\frac{k^2}{A^2B^2}, \quad E_3^3=E_4^4=-\frac{1}{8\pi}\frac{k^2}{A^2B^2}. \tag{8}$$

The particle density of the configuration is also given by

$$\rho=\rho_p+\lambda. \tag{9}$$

The Einstein's field equations

$$R_i^j-\frac{1}{2}Rg_i^j=-8\pi T_i^j, \tag{10}$$

for the metric (1) lead to

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}=-8\pi p-\frac{k^2}{A^2B^2}, \tag{11}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}=-8\pi p-\frac{k^2}{A^2B^2}, \tag{12}$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-8\pi(p-\lambda)+\frac{k^2}{A^2B^2}, \tag{13}$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{A}\dot{C}}{AC}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{A^2}=8\pi\rho+\frac{k^2}{A^2B^2}, \tag{14}$$

$$\alpha\left(\frac{\dot{A}}{A}-\frac{\dot{B}}{B}\right)=0. \tag{15}$$

The expansion scalar θ and the shear scalar σ are given by

$$\theta=u^i_{;i}=2\frac{\dot{A}}{A}+\frac{\dot{C}}{C}, \tag{16}$$

$$\sigma^2=\frac{1}{2}\sigma_{ij}\sigma^{ij}=\frac{1}{3}\left(\frac{\dot{C}}{C}-\frac{\dot{A}}{A}\right)^2, \tag{17}$$

where an overdot stands for the first and double overdot for the second derivative with respect to t.

3 Solution of the Field Equations

From (15) we get

$$A = mB. \tag{18}$$

For simplicity and with out loss of generally, we assume $m = 1$.

The field equations (11)–(14) are a system of three equations with five unknown parameters A, C, p, ρ, λ as (11) and (12) are same under condition given by (18). two additional constraints relating these parameters are required to obtain explicit solutions of the system. We assume that the expansion θ in the model is proportional to the shear σ. This condition leads to

$$A = C^n, \tag{19}$$

where n is a constant.

From (11)–(14) we get

$$8\pi\rho = 2\frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} - \frac{k^2}{A^2B^2}, \tag{20}$$

$$8\pi\lambda = \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} + \frac{\dot{A}^2}{A^2} - \frac{\alpha^2}{A^2} - 2\frac{k^2}{A^2B^2}. \tag{21}$$

The highly non-linear field equations can be solved for the following two types of geometric strings

3.1 Case I (Reddy String)

In this case

$$\rho + \lambda = 0. \tag{22}$$

From (20) and (21), we obtain

$$8\pi(\rho + \lambda) = \frac{\ddot{A}}{A} - \frac{\ddot{C}}{C} + 3\frac{\dot{A}\dot{C}}{AC} + 2\frac{\dot{A}^2}{A^2} - 2\frac{\alpha^2}{A^2} - 3\frac{k^2}{A^2B^2}. \tag{23}$$

By using (18) and (19), (23) can be written as

$$2\ddot{C} + 2\frac{(3n^2 + 2n)}{(n-1)}\frac{\dot{C}^2}{C} = \frac{6k^2}{(n-1)}C^{1-4n} + \frac{4\alpha^2}{(n-1)}C^{1-2n}. \tag{24}$$

Let $\dot{C} = f(C)$ which implies that $\ddot{C} = ff'$, where $f' = \frac{df}{dC}$. Therefore (24) reduces to

$$\frac{d}{dC}(f^2) + \gamma\frac{f^2}{C} = \frac{6k^2}{(n-1)}C^{1-4n} + \frac{4\alpha^2}{(n-1)}C^{1-2n}, \tag{25}$$

where, $\gamma = 2\frac{(3n^2+2n)}{(n-1)}$. Equation (25), after integration, reduces to

$$f^2 = \frac{6k^2}{(n-1)(\gamma - 4n + 2)}C^{2(1-2n)} + \frac{4\alpha^2}{(n-1)(\gamma - 2n + 2)}C^{2(1-n)} + l_0 C^{-\gamma}, \tag{26}$$

where l_0 is an integrating constant. To get deterministic solution in terms of cosmic time t, we suppose $l_0 = 0$. In this case (26) takes the form

$$f^2 = a^2 C^{-2(2n-1)} + b^2 C^{2(1-n)}, \tag{27}$$

where

$$a^2 = \frac{6k^2}{(n-1)(\gamma - 4n + 2)}, \quad b^2 = \frac{4\alpha^2}{(n-1)(\gamma - 2n + 2)}. \tag{28}$$

Therefore, we have

$$\frac{C^{2n-1} dC}{\sqrt{a^2 + b^2 C^{2n}}} = dt. \tag{29}$$

In this case integrating (27), we obtain

$$C^2 = \left(\frac{n^2 b^4 t^2 - a^2}{b^2}\right)^{\frac{1}{n}}. \tag{30}$$

Hence, we have

$$A^2 = B^2 = \left(\frac{n^2 b^4 t^2 - a^2}{b^2}\right). \tag{31}$$

Thus the metric (1) reduces to the form

$$ds^2 = -dt^2 + \left(\frac{n^2 b^4 t^2 - a^2}{b^2}\right) dx^2 + \left(\frac{n^2 b^4 t^2 - a^2}{b^2}\right) e^{-2\alpha x} dy^2 + \left(\frac{n^2 b^4 t^2 - a^2}{b^2}\right)^{\frac{1}{n}} dz^2. \tag{32}$$

After using suitable transformation

$$\frac{x}{b} = X, \qquad \frac{y}{b} = Y, \qquad \frac{z}{b^{\frac{1}{n}}} = Z, \qquad \beta t = T, \tag{33}$$

the metric (32) reduces to

$$ds^2 = -\frac{a}{nb^2} dT^2 + (T^2 - a^2) dX^2 + (T^2 - a^2) e^{-2\alpha x} dY^2 + (T^2 - a^2)^{\frac{1}{n}} dZ^2, \tag{34}$$

where $\beta = nb^2$.

3.1.1 The Geometric and Physical Significance of Model

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (34) are given by

$$8\pi p = -\frac{1}{(T^2 - a^2)^2} \left[\left(\frac{n^3 - n + 1}{n^2}\right) T^2 + k^2 + b^4 \right], \tag{35}$$

$$8\pi \rho = \frac{1}{(T^2 - a^2)^2} \left[\left(\frac{n+2}{n} - \alpha^2 b^2\right) T^2 - k^2 b^4 + \alpha^2 b^2 a^2 \right], \tag{36}$$

$$8\pi\lambda = \frac{1}{(T^2-a^2)^2}\left[\left(\alpha^2 b^2 - \frac{n+2}{n}\right)T^2 + k^2 b^4 - \alpha^2 b^2 a^2\right], \tag{37}$$

$$8\pi\rho_p = 2\frac{1}{(T^2-a^2)^2}\left[\left(\frac{n+2}{n} - \alpha^2 b^2\right)T^2 - k^2 b^4 + 2\alpha^2 b^2 a^2\right], \tag{38}$$

$$\theta = \frac{(2n+1)}{n}\frac{T}{T^2-a^2}, \tag{39}$$

$$\sigma^2 = \frac{1}{3}\left(\frac{1-n}{n}\right)^2 \frac{T^2}{(T^2-a^2)^2}, \tag{40}$$

$$V^3 = \sqrt{-g} = e^{\frac{-\alpha b}{a}x}\left(\frac{T^2-a^2}{b^2}\right)^{\frac{2n+1}{n}}. \tag{41}$$

From (39) and (40), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} = \text{constant}. \tag{42}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -1 - \frac{3n}{2n+1}\frac{T^2-a^2}{T^2} + \frac{6}{2n+1}, \tag{43}$$

where $R^3 = A^2 C$ is the spatial average scalar factor. From (43), we observe that

$$q < 0 \quad \text{if } T^2 > \frac{3na^2}{5(n-1)}, \tag{44}$$

and

$$q > 0 \quad \text{if } T^2 < \frac{3na^2}{5(n-1)} \tag{45}$$

also for the condition

$$T = \sqrt{\frac{na^2}{n-2}}, \tag{46}$$

$q = -1$ as in the case of de-sitter universe. The energy conditions, $\rho_p \geq 0$ and $\rho \geq 0$ lead to

$$T^2 \geq n\frac{k^2 b^4 - \alpha^2 b^2 a^2}{n - n\alpha^2 b^2 + 2}. \tag{47}$$

The model (3) starts with a big bang at $T = a$. The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta} = \text{constant}$, hence the model does not approach isotropy. There is a Point Type singularity (MacCallum [74]) in the model at $T = a$. For the condition $T^2 > \frac{3na^2}{5(n-1)}$, the solution gives accelerating model of the universe whereas when $T^2 < \frac{3na^2}{5(n-1)}$, our solution represents decelerating model of the universe.

According to Refs. [1] and [19], when $\frac{\rho_p}{|\lambda|} > 1$, in the process of evolution, the universe is dominated by massive strings, and when $\frac{\rho_p}{|\lambda|} < 1$, the universe is dominated by the strings.

In this case from (37) and (38), we obtain

$$\frac{\rho_p}{|\lambda|} = 2 > 1. \tag{48}$$

Thus, in our model, the universe is dominated by massive strings throughout the whole process of evolution.

3.1.2 Solutions in Absence of Magnetic Field

In absence of magnetic field, i.e. when $a \to 0$ i.e. $K \to 0$, we obtain

$$C^2 = (n\beta t^2)^{\frac{1}{n}}, \tag{49}$$

and

$$A^2 = B^2 = n\beta t^2. \tag{50}$$

Hence, in this case, the geometry of the universe (34) reduces to

$$ds^2 = -dt^2 + (n\beta t^2)dx^2 + (n\beta t^2)e^{-2\alpha x}dy^2 + (n\beta t^2)^{\frac{1}{n}}dz^2, \tag{51}$$

which after using a suitable transformation of coordinates

$$x = X, \qquad y = Y, \qquad z = Z, \qquad \sqrt{n\beta}t = T, \tag{52}$$

the model (51) reduces to

$$ds^2 = -\frac{dT^2}{(nb)^2} + T^2dX^2 + T^2e^{-2\alpha X}dY^2 + T^{\frac{2}{n}}dZ^2. \tag{53}$$

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (53) are given by

$$8\pi p = -2\left(\frac{n^2+n+4}{n^2}\right)\frac{1}{T^2}, \tag{54}$$

$$8\pi\rho = 4\left(1+\frac{2}{n}\right)\frac{1}{T^2} - \frac{\alpha^2}{T^4}, \tag{55}$$

$$8\pi\lambda = \frac{\alpha^2}{T^4} - 4\left(1+\frac{2}{n}\right)\frac{1}{T^2}, \tag{56}$$

$$8\pi\rho_p = 8\left(1+\frac{2}{n}\right)\frac{1}{T^2} - 2\frac{\alpha^2}{T^4}, \tag{57}$$

$$\theta = 2\left(\frac{2n+1}{n}\right)\frac{1}{T}, \tag{58}$$

$$\sigma^2 = \frac{4}{3}\left(\frac{1}{n^2} - 1\right)\frac{1}{T^2}, \tag{59}$$

$$V^3 = e^{-2\alpha X}T^{2(\frac{2n+1}{n})}. \tag{60}$$

From (58) and (59), we get

$$\frac{\sigma^2}{\theta^2} = \text{constant}. \tag{61}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = -1 + \frac{3n}{2(2n+1)}. \tag{62}$$

From (62), we observe that

$$q < 0 \quad \text{if } n > -2, \tag{63}$$

and

$$q > 0 \quad \text{if } n < -2. \tag{64}$$

The energy conditions, $\rho_p \geq 0$ and $\rho \geq 0$ lead to

$$T \geq \frac{\alpha^2}{4(1+\frac{2}{n})}. \tag{65}$$

The model (53) starts with a big bang at $T = 0$. The expansion in the model decreases as time increases. The proper volume of the model increases as time increases. Since $\frac{\sigma}{\theta} =$ constant, hence the model does not approach isotropy. There is a Point Type singularity (MacCallum [74]) in the model at $T = 0$. For the $n > -2$, the solution gives accelerating model of the universe whereas for $n < -2$, our solution represents decelerating model of the universe. We also see that for the condition $T^2 < \frac{n\alpha^2}{4(n+1)}$, $\lambda > 0$, and for $T^2 > \frac{n\alpha^2}{4(n+1)}$, $\lambda < 0$.

From (56) and (57) we obtain

$$\frac{\rho_p}{|\lambda|} = 2 > 1. \tag{66}$$

Therefore, in this case, the universe is dominated by massive strings throughout the whole process of evolution.

3.2 Case II (Nambu or Geometric String)

In this case

$$\rho - \lambda = 0. \tag{67}$$

From (20) and (21) we obtain

$$8\pi(\rho - \lambda) = \frac{\ddot{C}}{C} - \frac{\ddot{A}}{A} + 3\frac{\dot{A}\dot{C}}{AC} + \frac{k^2}{A^2B^2}. \tag{68}$$

By using (18) and (19), (68) can be written as

$$2\ddot{C} + 2\left(\frac{n^2-4n}{n-1}\right)\frac{\dot{C}^2}{C} = \frac{2k^2}{n-1}C^{1-4n}. \tag{69}$$

Let $\dot{C} = f(C)$ which implies that $\ddot{C} = ff'$, where $f' = \frac{df}{dC}$. Therefore (69) takes the form

$$\frac{d}{dC}(f^2) + \xi\frac{f^2}{C} = \frac{2k^2}{n-1}C^{1-4n}, \tag{70}$$

which on integrating leads to

$$dt = \frac{dC}{\sqrt{[-\frac{2k^2}{2n^2+3n+1}C^{2(1-2n)} + lC^{-2(\frac{n^2-4n}{n-1})}]}}, \tag{71}$$

where l is a positive integrating constant. Hence the model (1) is reduced to

$$ds^2 = -\frac{dC^2}{[-\frac{2k^2}{2n^2+3n+1}C^{2(1-2n)} + lC^{-2(\frac{n^2-4n}{n-1})}]} + C^{2n}dx^2 + C^{2n}e^{-2\alpha x}dy^2 + C^2dz^2. \tag{72}$$

After using suitable transformation the model (72) reduces to

$$ds^2 = -\frac{dT^2}{[-\frac{2k^2}{2n^2+3n+1}T^{2(1-2n)} + lT^{-2(\frac{n^2-4n}{n-1})}]} + T^{2n}dx^2 + T^{2n}e^{-2\alpha x}dy^2 + T^2dz^2. \tag{73}$$

3.2.1 The Geometric and Physical Significance of Model

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (73) are given by

$$8\pi p = -[n^2 - 2(n+1)]lT^{-2(\frac{n^2-3n-1}{n-1})} + 2\left(\frac{3n^2+2n-2}{2n^2+3n+1}\right)k^2T^{-4n}, \tag{74}$$

$$8\pi\lambda = 8\pi\rho = (n^2+2n)lT^{-2(\frac{n^2-3n-1}{n-1})} - 2\left(\frac{3n^2+5n+1}{2n^2+3n+1}\right)k^2T^{-4n} - \alpha^2T^{-2n}, \tag{75}$$

$$\rho_p = 0, \tag{76}$$

$$\theta = (2n+1)\left[lT^{-2(\frac{n^2-3n-1}{n-1})} - 2\frac{k^2}{2n^2+3n+1}T^{-4n}\right]^{\frac{1}{2}}, \tag{77}$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left[lT^{-2(\frac{n^2-3n-1}{n-1})} - 2\frac{k^2}{2n^2+3n+1}T^{-4n}\right], \tag{78}$$

$$V^3 = \sqrt{-g} = e^{-\alpha x}T^{2n+1}. \tag{79}$$

From (77) and (78), we obtain

$$\frac{\sigma^2}{\theta^2} = \frac{(n-1)^2}{3(n+2)^2} = \text{constant}. \tag{80}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = \left(\frac{3}{2n+1}\right)\left[\frac{lT^{-2(\frac{n^2-3n-1}{n-1})} - 2(\frac{4n-3}{2n^2+3n+1})k^2T^{-4n}}{lT^{-2(\frac{n^2-3n-1}{n-1})} - 2(\frac{k^2}{2n^2+3n+1})T^{-4n}}\right]. \tag{81}$$

From (81), we observe that

$$q<0 \quad \text{if } T^{2(\frac{n^2+n+1}{n-1})} < \frac{2(4n-3)}{l(2n^2+3n+1)}, \tag{82}$$

and

$$q>0 \quad \text{if } T^{2(\frac{n^2+n+1}{n-1})} > \frac{2(4n-3)}{l(2n^2+3n+1)}. \tag{83}$$

Also we see that for the condition

$$T^{\frac{n^2+n+1}{n-1}} = \sqrt{\frac{14n-8}{l(2n^2+3n+1)(n+2)}}, \tag{84}$$

$q=-1$ as in the case of de-sitter universe. The energy condition $\rho \geq 0$ leads to

$$T^{2(\frac{2n+1}{n-1})} \geq \frac{1}{l(n^2+2n)}\left[2\left(\frac{3n^2+5n+2}{2n^2+3n+1}\right)k^2T^{-2n}-\alpha^2\right]. \tag{85}$$

The model (73) starts with a big bang at $T=0$. The expansion in the model decreases as time increases. The expansion in the model stops at $T=\infty$. When $T\to 0$ then $\rho\to\infty$, $\lambda\to\infty$. When $T\to\infty$ then $\rho\to 0$, $\lambda\to 0$. Also $p\to\infty$ when $T\to 0$ and $p\to 0$ when $T\to\infty$. Since $\frac{\sigma}{\theta}=$ constant, the model does not approach isotropy. There is a Point Type singularity (MacCallum [74]) in the model at $T=0$. For the condition $T^{2(\frac{n^2+n+1}{n-1})} < \frac{2(4n-3)}{l(2n^2+3n+1)}$, the solution gives accelerating model of the universe whereas when $T^{2(\frac{n^2+n+1}{n-1})} > \frac{2(4n-3)}{l(2n^2+3n+1)}$, our solution represents decelerating model of the universe.

From (55) and (56), we obtain

$$\frac{\rho_p}{|\lambda|}=0. \tag{86}$$

Hence, in this case the strings dominate over the particles.

3.2.2 *Solutions in Absence of Magnetic Field*

In absence of magnetic field, i.e. when $K\to 0$, the metric (73) reduces to

$$ds^2=-\frac{dC^2}{[lC^{-2(\frac{n^2-4n}{n-1})}]}+T^{2n}dx^2+T^{2n}e^{-2\alpha x}dy^2+T^2dz^2. \tag{87}$$

The pressure (p), the energy density (ρ), the string tension (λ), the particle density (ρ_p), the scalar of expansion (θ), the shear (σ) and the proper volume (V^3) for the model (87) are given by

$$8\pi p=-[n^2-2(n+1)]lT^{-2(\frac{n^2-3n-1}{n-1})}, \tag{88}$$

$$8\pi\lambda=8\pi\rho=(n^2+2n)lT^{-2(\frac{n^2-3n-1}{n-1})}-\alpha^2T^{-2n}, \tag{89}$$

$$\rho_p=0, \tag{90}$$

$$\theta=(2n+1)\left[lT^{-2(\frac{n^2-3n-1}{n-1})}\right]^{\frac{1}{2}}, \tag{91}$$

$$\sigma^2 = \frac{(n-1)^2}{3}\left[lT^{-2(\frac{n^2-3n-1}{n-1})}\right], \tag{92}$$

$$V^3 = \sqrt{-g} = e^{-\alpha x} T^{2n+1}. \tag{93}$$

From (91) and (92), we obtain

$$\frac{\sigma^2}{\theta^2} = \text{constant}. \tag{94}$$

The deceleration parameter is given by

$$q = -\frac{\ddot{R}R}{\dot{R}^2} = \left(\frac{3}{2n+1}\right). \tag{95}$$

From (95), we observe that

$$q < 0 \quad \text{if } n < -\frac{1}{2}, \tag{96}$$

and

$$q > 0 \quad \text{if } n > -\frac{1}{2}. \tag{97}$$

Also we see that for

$$n = -2, \tag{98}$$

$q = -1$ as in the case of de-sitter universe. The energy condition, $\rho \geq 0$ leads to

$$T^{2(\frac{2n+1}{n-1})} \geq \frac{\alpha^2}{l(n^2+2n)}. \tag{99}$$

The model (85) starts with a big bang at $T = 0$. The expansion in the model decreases as time increases. The expansion in the model stops at $T = \infty$. When $T \to 0$ then $\rho \to \infty$, $\lambda \to \infty$. When $T \to \infty$ then $\rho \to 0$, $\lambda \to 0$. Also $p \to \infty$ when $T \to 0$ and $p \to 0$ when $T \to \infty$. Since $\frac{\sigma}{\theta}$ = constant, the model does not approach isotropy. There is a Point Type singularity (MacCallum [74]) in the model at $T = 0$. For $n < -\frac{1}{2}$ the solution gives accelerating model of the universe whereas when $n > -\frac{1}{2}$, our solution represents decelerating model of the universe.

4 Conclusion

In this paper we have presented a new solution of Einstein's field equations for Bianchi type-III space-time with a cloud of strings in presence of perfect fluid and electromagnetic field. We have considered two cases (i) Reddy String and (ii) Nambu String in presence and absence of magnetic field. In the first case our model starts with a big bang at $T = a$ in presence of magnetic field whereas in the absence of magnetic field the model has a big bang singularity at $T = 0$. In the case (ii) the models have big bang singularity at $T = 0$ in presence and absence of magnetic field. In both cases our models are in accelerating phase under appropriate conditions. In case (i) the universe is dominated by massive strings throughout the whole process of evolution in presence and absence of magnetic field. But it is observed that in case (ii) the string dominates over the particle. In both cases the models do not approach isotropy. Our models are realistic and new to the others.

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